

# The disappearing momentum of the supercurrent in the superconductor to normal phase transformation

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A superconductor in a magnetic field has surface currents that prevent the magnetic field from penetrating its interior. These currents carry kinetic energy and mechanical momentum. When the temperature is raised and the system becomes normal the currents disappear. Where do the kinetic energy and mechanical momentum of the currents go, and how? Here we propose that the answer to this question reveals a key necessary condition for materials to be superconductors, that is not part of conventional BCS-London theory: *superconducting materials need to have hole carriers*.

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A superconductor in a magnetic field has shielding currents that keep magnetic field lines out of the superconductor except within a surface layer of thickness  $\lambda_L$ , the London penetration depth. It was discovered in 1914 by Kammerlingh Onnes [1] that when the magnetic field exceeds a critical value  $H_c$  that depends on temperature, the system becomes normal and the shielding currents disappear. In this paper we discuss what happens to the kinetic energy and mechanical momentum of the shielding currents when the system becomes normal, and how it happens, and argue that it has fundamental unrecognized implications for the understanding of superconductivity. We discuss only type I superconductors.

Until the discovery of the Meissner effect in 1933 [2] it was generally believed that superconductors were nothing more than ‘perfect conductors’ with zero resistivity. Within this point of view, when superconductors in the presence of a magnetic field became normal by raising the temperature above the critical temperature for the given applied field, or by raising the applied field above the critical field for the given temperature, the resistivity would become finite and the shielding currents would decay by the usual scattering processes in normal metals, i.e. phonons and impurities. This would cause the kinetic energy of the shielding currents to be dissipated as Joule heat in an irreversible way, and the mechanical momentum of the shielding currents to be transferred to the body as a whole through the same scattering processes that dissipate the energy and bring the current to a halt. Within this point of view, if subsequently the system would be cooled again it was expected that the shielding currents would not reappear, rather that the magnetic field would remain in the interior of the body as it became superconducting again.

Meissner and Ochsenfeld’s 1933 discovery [2] however showed that on lowering the temperature the shielding currents are restored and the magnetic field is expelled. This suggested (but did not prove) that the kinetic energy of the shielding currents was in fact not lost to Joule heat as the system became normal, but rather became stored somewhere where it could be subsequently retrieved and used to propel the shielding currents when the system was cooled again. Indeed very precise exper-

iments by Keesom and coworkers [3] showed that in the process of the system becoming normal and the shielding currents decaying to zero *no* irreversible Joule heating occurs. The kinetic energy of the supercurrents is used up in paying for the difference in free energies between normal and superconducting states, as first discussed by H. London [4].

A conundrum that didn’t exist before was thus created by Meissner’s discovery: if there are no collision processes that dissipate Joule heat in the superconductor-to-normal transition in the presence of a magnetic field, what happens to the mechanical momentum of the disappearing current? The kinetic energy of the current is ‘stored’ in the normal state electronic state, but its momentum is not. Of course the only possible answer is that the momentum of the current is transmitted to the body as a whole. But what is the physical mechanism by which this transfer of momentum happens with no energy transfer and no energy dissipation? Surprisingly this basic and fundamental question has *never* been asked (nor answered) in the extensive literature on superconductivity since 1933 (213,616 papers according to the Web of Science).

How do we actually know that the shielding supercurrents carry mechanical momentum? Because it is expected theoretically and has been verified experimentally by measuring the gyromagnetic effect in superconductors [5]: upon applying a magnetic field to a spherical or cylindrical superconductor hanging from a thread, shielding currents develop *and* the body as a whole starts to rotate to keep the total angular momentum zero. The measured angular momentum of the body as a whole corresponds precisely to what is expected if the mechanical momentum density of the shielding current  $\vec{\mathcal{P}}$  is given by

$$\vec{\mathcal{P}} = \frac{m_e}{e} \vec{J} \quad (1)$$

where  $\vec{J}$  is the current density,  $m_e$  the bare electron mass and  $e$  ( $< 0$ ) the electron charge. For applied magnetic field  $H$ ,  $J = c/(4\pi\lambda_L)H$ , with  $\lambda_L$  the London penetration depth.

The total momentum of the shielding currents will be

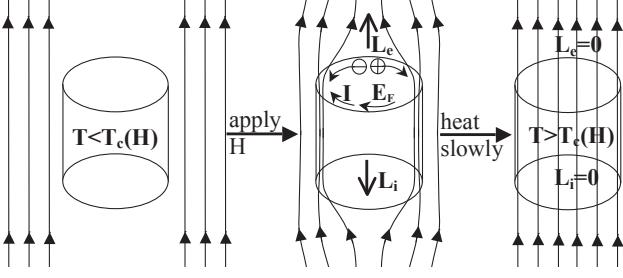


FIG. 1: When a magnetic field is applied to the superconductor, a Faraday field  $E_F$  is generated that prevents penetration of the magnetic field beyond a London penetration depth of the surface of the superconductor.  $E_F$  is clockwise as seen from the top in the Figure.  $E_F$  drives electrons in the supercurrent to flow counterclockwise giving rise to a clockwise current that generates a magnetic field opposite to the applied one, and  $E_F$  also drives the positive ions in the body to move clockwise causing the body to rotate rigidly clockwise. The total angular momentum of electrons plus ions  $\vec{L}_e + \vec{L}_i$  is zero at all times. When the temperature is raised above  $T_c(H)$ , the system becomes normal, the magnetic field penetrates the interior, the supercurrent stops flowing and the body stops rotating. The Faraday field generated in this latter process points in the same clockwise direction as the one generated when the field is applied.

zero, but the angular momentum will not. It is given by

$$\vec{L}_e = \int d^3r \vec{r} \times \vec{P}(\vec{r}) \quad (2)$$

and an opposite angular momentum has to be acquired by the body as a whole. If we envision a process where the superconductor is initially at rest without magnetic field, application of a magnetic field will both induce the shielding currents with their angular momentum  $\vec{L}_e$  and impart opposite angular momentum to the body as a whole  $\vec{L}_i = -\vec{L}_e$ . Both processes can be simply understood as arising from the force imparted by the Faraday electric field induced as the magnetic field is applied, counterclockwise for the negative electrons in the shielding currents and clockwise for the positive ions in the body as seen from the direction where the magnetic field points, as shown schematically in Fig. 1. The total angular momentum of the system (electrons plus ions) remains zero if the system is charge neutral.

As we subsequently slowly raise the temperature and the system becomes normal, the shielding currents stop and the rotation of the body has to stop also, so that the total angular momentum remains zero, now with  $\vec{L}_e = \vec{L}_i = 0$ . This *cannot* be understood as arising from force imparted by the Faraday electric field generated as the magnetic field lines penetrate the body. Quite the contrary, the Faraday electric field acts in the same clockwise direction as when the magnetic field was first applied, trying to restore both the rotation of the body and the flow of shielding currents. How then does the body stop rotating?

There is no microscopic theory that describes the process of the superconductor-normal transition in the presence of a magnetic field (nor the reverse transition) within the conventional theory of superconductivity. These problems have been studied using the phenomenological time-dependent Ginzburg-Landau formalism [6–9]. Within this formalism Eilenberger has shown [10] that when the superfluid electron density decreases its mechanical momentum is transferred to the normal electrons, and according to Eilenberger “this momentum then decays with the transport relaxation time  $\tau$ ”. Clearly this cannot be correct since it would lead to irreversible Joule heating which is not observed [3]. *How then do electrons in the supercurrent transfer their mechanical momentum to the ionic lattice without energy dissipation?*

Consider Bloch electrons in the weak binding approximation moving in a perfect crystal. Electrons interact with the crystal potential through its Fourier components  $U_{\vec{K}}$  where  $\vec{K}$  are reciprocal lattice vector. Electrons near the bottom of the band are only weakly affected by the lattice potential, since the energy of an electron scattered from  $\vec{k}$  to  $\vec{k} \pm \vec{K}$  will be vastly higher than  $\epsilon_k^0$ , the free electron energy for  $\vec{k}$  near the bottom of the band. Instead, electrons near the top of the band are strongly affected by the lattice potential since the energies  $\epsilon_k^0$  and  $\epsilon_{k \pm K}^0$  will be nearly equal for some reciprocal lattice vector(s)  $\vec{K}$ . In affecting the electronic state, the lattice transfers momentum to the electron. By Newton’s third law, the electron transfers momentum to the lattice. This indicates that the electrons that are most effective in transferring mechanical momentum to the lattice without energy dissipation are *electrons near the top of the band*. Since superconducting electrons becoming normal in the presence of a magnetic field need to transfer mechanical momentum to the lattice without energy dissipation, we conclude that *materials that can become superconductors need to have electrons near the top of electronic energy bands*. In other words, *holes*.

More generally, consider the semiclassical equation of motion for an electron of wavevector  $\vec{k}$ :

$$\hbar \frac{d\vec{k}}{dt} = \vec{F}_{ext} \quad (3)$$

where  $\vec{F}_{ext}$  is an external force. The *total* force acting on the electron is

$$m_e \frac{d\vec{v}_k}{dt} = \vec{F}_{ext} + \vec{F}_L \quad (4)$$

with

$$\vec{v}_k = \frac{1}{\hbar} \frac{\partial \epsilon_k}{\partial \vec{k}} \quad (5)$$

and  $\vec{F}_L$  the force that the lattice exerts on the electron, given by

$$\vec{F}_L = (m_e \frac{1}{\hbar^2} \frac{\partial^2 \epsilon_k}{\partial \vec{k} \partial \vec{k}} - 1) \vec{F}_{ext}. \quad (6)$$

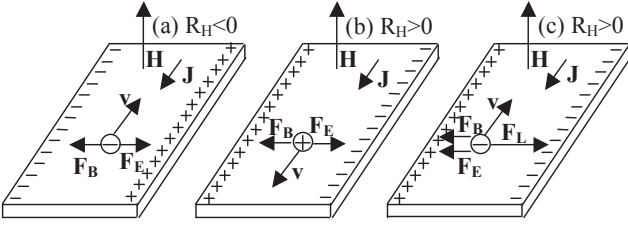


FIG. 2: (a) Hall effect when carriers are electron-like. Electric and magnetic forces  $F_E$  and  $F_B$  on the charge carriers in direction perpendicular to the current  $\vec{J}$  point in opposite directions and cancel each other. (b) Hall effect when carriers are hole-like. Electric and magnetic force on *positive* charge carriers (holes) cancel each other. (c) Reinterpretation of the forces for case (b): since the mobile charge carriers are always electrons, electric and magnetic forces point in the same direction and need to be cancelled by the force  $F_L$  exerted by the lattice on electrons.

By Newton's third law, the electron in turn exerts a force on the lattice

$$\vec{F}_{on-L} = -\vec{F}_L = (1 - m_e \frac{1}{\hbar^2} \frac{\partial^2 \epsilon_{\vec{k}}}{\partial k \partial k}) \vec{F}_{ext}. \quad (7)$$

which transfers momentum from the electron to the lattice. The largest momentum transfer will occur when the second derivative term in Eq. (7) is negative, which happens when electrons are near the top of a band, i.e. when there is *hole* conduction.

Consider next the motion of electrons in crossed electric and magnetic fields, as shown in Fig. 2. The Hall coefficient is defined as  $R_H = E_y / (J_x H)$ , with  $J_x$  the current in the longitudinal direction,  $E_y$  the electric field in the transverse direction and  $H$  the magnetic field in the perpendicular direction. By setting  $J_x = nev$ , with  $n$  the concentration of carriers of charge  $e$  moving with drift velocity  $v$ , electric and magnetic Lorentz forces  $F_E$  and  $F_B$  are balanced for  $E_y = (v/c)H$  ( $F_E = eE_y = F_B = (ev/c)H$ ) and it follows that the Hall coefficient is given by

$$R_H = \frac{1}{nec} \quad (8)$$

with the sign as shown in Fig. 2(a) assuming the mobile carriers are electrons. As shown by Ashcroft and Mermin [11], Eq. (8) holds for Bloch electrons, with the current and number of carriers given by

$$\vec{J} = \int_{occ} \frac{d^3k}{4\pi^3} \frac{1}{\hbar} \frac{\partial \epsilon_{\vec{k}}}{\partial \vec{k}} \quad (9a)$$

$$n = \int_{occ} \frac{d^3k}{4\pi^3} \quad (9b)$$

if all occupied  $k$ -space orbits are closed, which occurs when the band is closed to empty. In this case, electric

and magnetic forces on electrons are balanced on average as shown in Fig. 2(a) and no net force is exerted by the lattice on electrons nor by electrons on the lattice as the current flows.

On the other hand, if all *unoccupied*  $k$ -space orbits are closed, which occurs when the band is almost full, the Hall voltage has opposite sign and the Hall coefficient is given by [11]

$$R_H = \frac{1}{n_h |e| c} \quad (10)$$

with

$$\vec{J} = \int_{unocc} \frac{d^3k}{4\pi^3} \frac{1}{\hbar} \frac{\partial \epsilon_{\vec{k}}}{\partial \vec{k}} \quad (11a)$$

$$n_h = \int_{unocc} \frac{d^3k}{4\pi^3} \quad (11b)$$

According to Fig. 2(b), the electric and magnetic forces *on holes* are equal and opposite and no net force results. However, this is misleading, since electric and magnetic forces act on electrons and not on holes. As shown in Fig. 2(c), electric and magnetic forces on *electrons* point in the same direction, and a lattice counterforce  $F_L = F_E + F_B$  is exerted by the lattice on the electron to keep its trajectory along the direction of the current. This in turn implies that when  $R_H > 0$  a *steady force*  $-F_L = -(F_E + F_B)$  is exerted by the electron on the lattice as the current flows.

This force exerted by the carriers on the lattice when current flows transfers momentum from the carriers to the lattice without energy dissipation. We are not aware of any other physical mechanism by which charge carriers in a solid can transfer momentum to the lattice without scattering processes that lead to energy dissipation. Consequently, we propose that the process shown in Fig. 2(c) describes the essential physics of how the momentum of the supercurrents is transferred to the body as a whole when a superconductor in a magnetic field makes a reversible transition to the normal state.

A detailed realization of this mechanism is provided by the theory of hole superconductivity [12]. Within that theory, superconducting electrons reside in mesoscopic orbits of radius  $2\lambda_L$  [13], with  $\lambda_L$  the London penetration depth. When carriers go from normal to superconducting, their orbits expand from radius  $k_F^{-1}$  to radius  $2\lambda_L$  driven by lowering of quantum kinetic energy, and if a magnetic field is present they acquire through the magnetic Lorentz force the angular velocity required to provide a dynamical explanation of the origin of the Meissner current [14]. We now explain how the transfer of momentum to the lattice occurs for the case of interest here, for a planar geometry for simplicity.

Figure 3 shows schematically the large orbits in the superconducting state (large overlapping circles) centered below the phase boundary line (horizontal dotted line),

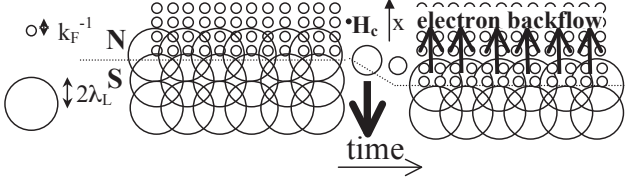


FIG. 3: Normal-superconductor phase boundary advancing in the  $-\hat{x}$  direction. Large orbits of superconducting electrons near the phase boundary extend into the normal region. As these orbits shrink, it causes a backflow of normal electrons flowing in the  $+\hat{x}$  direction as indicated by the vertical arrows. Magnetic field  $H_c$  points out of the paper.

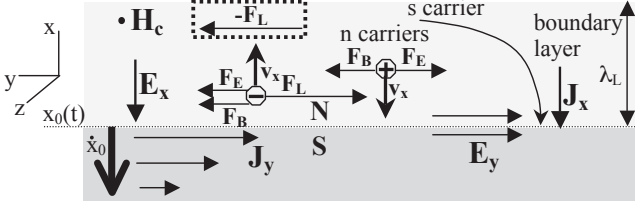


FIG. 4: Schematics of currents and fields for the situation depicted in Fig. 3 where the normal phase is advancing in the  $-\hat{x}$  direction. The Faraday field  $E_y$  points in the negative  $\hat{y}$  direction. The normal electron backflow depicted in the positive  $\hat{x}$  direction corresponds to a normal current  $J_x$  flowing in the  $-\hat{x}$  direction within a boundary layer of thickness  $\lambda_L$  from the phase boundary. If the normal carriers ( $n$  carriers) are hole-like, the lattice exerts force  $F_L$  on the carriers in the  $\hat{y}$  direction and correspondingly the carriers exert a force  $-F_L$  on the lattice (shown in the dotted rectangle) in the  $+\hat{x}$  direction. The momentum in the  $\hat{y}$  direction imparted by the backflow on the lattice equals the momentum in the  $\hat{y}$  direction of the carriers of the supercurrent  $J_y$  becoming normal that is lost through the action of the magnetic Lorentz force as the orbits shrink.

and the small orbits (small nonoverlapping circles) in the normal state above the phase boundary line, in the presence of the critical magnetic field  $H_c$  in the normal region pointing out of the paper. As the phase boundary moves down into the superconducting region, large orbits right at the phase boundary shrink, as shown by the circles of diminishing radius. This causes negative charge to be transferred out of a boundary layer of thickness  $\lambda_L$  above the phase boundary, and gives rise to a *backflow* of electrons moving in the positive  $\hat{x}$  direction [15], indicated by the vertical arrows labeled ‘electron backflow’. This corresponds equivalently to a current flowing in the  $-\hat{x}$  direction.

Figure 4 shows all the currents and fields schematically, in a situation where the phase boundary located at  $x_0(t)$  is moving down into the superconducting phase at a uniform speed  $\dot{x}_0$ . A Faraday electric field  $E_y$  pointing in the  $-\hat{y}$  direction is generated at and in the neighborhood of the phase boundary due to the changing magnetic flux,

given by

$$E_y = \frac{\dot{x}_0}{c} H_c \quad (12)$$

The backflow current  $J_x$  flowing in the  $-\hat{x}$  direction is assumed to be hole-like, corresponding to the situation in Figure 2(c), and has magnitude  $J_x = n_h |e| \dot{x}_0$ , with  $n_h$  the hole carrier concentration. The forces on electrons are balanced by a force  $F_L$  exerted by the lattice on the electrons, and the electrons exert a counterforce on the lattice  $-F_L = -(F_E + F_B)$  in the  $+\hat{y}$  direction, shown in Fig. 4 in the dotted rectangle. In addition, the Faraday field exerts a force  $F_E$  on the lattice in the negative  $\hat{y}$  direction (not shown in Fig. 4). The net force on the lattice per carrier is then  $-F_E = eE_y$ , pointing in the  $+\hat{y}$  direction.

In more detail the balance is as follows. The superconducting electrons at the boundary have velocity in the  $+\hat{y}$  direction [15]

$$\vec{v}_y = \frac{e}{m_e c} \lambda_L H_c \hat{y} \quad (13)$$

and kinetic energy

$$\epsilon_{kin} = \frac{1}{2} m_e v_y^2 = \frac{e^2 \lambda_L^2}{2 m_e c^2} H_c^2 = \frac{1}{n_s} \frac{H_c^2}{8\pi} \quad (14)$$

using that  $1/\lambda_L^2 = 4\pi n_s e^2 / m_e c^2$ . An electron going from superconducting to normal shrinking its orbit effectively moves at a high speed  $v_x$  in the negative  $\hat{x}$  direction a distance  $\lambda_L$  in time  $\lambda_L / v_x$  under the action of the magnetic Lorentz force  $(e/c) v_x H_c$ , thereby changing its momentum by

$$\Delta p_y = \frac{e}{c} \lambda_L H_c. \quad (15)$$

This change in momentum is in the  $-\hat{y}$  direction, and exactly cancels the momentum in the  $+\hat{y}$  direction that the electron had initially being part of the Meissner current. This assumes that the speed  $v_x$  is much larger than  $\dot{x}_0$ , so that the effect of  $E_y$  over this short time (which applies a force in the opposite  $(+\hat{y})$  direction) can be neglected. This then explains how the Meissner current comes to a halt without dissipation. The kinetic energy that the electron lost Eq. (14) is the condensation energy per electron, i.e. what it costs to bring the electron from the superconducting to the normal state. Multiplying Eq. (14) by the number of superfluid carriers per unit volume  $n_s$  yields the condensation energy per unit volume  $H_c^2 / (8\pi)$  [4].

At the same time the ‘backflow’ normal electrons move at speed  $\dot{x}_0$  in the  $+\hat{x}$  direction, and traverse the boundary layer distance  $\lambda_L$  in time  $\Delta t = \lambda_L / \dot{x}_0$ . The *net* momentum imparted to the lattice in this process is  $\Delta p_y = F_E \Delta t = e E_y \Delta t = (e/c) \lambda_L H_c$  in the  $+\hat{y}$  direction, the same momentum that a superconducting electron becoming normal lost, Eq. (15). In this way the momentum of the supercurrent is transmitted to the lattice without



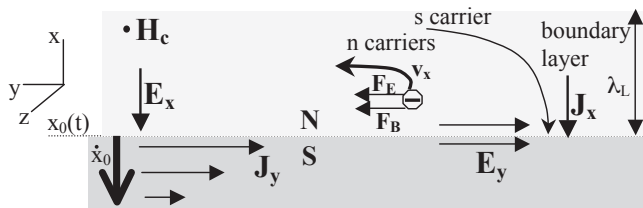


FIG. 5: Same as Fig. 4 if the normal state carriers are electrons. There is no force by the lattice on the backflow electrons hence the force in the  $\hat{y}$  direction is not balanced. A tangential flow of normal electrons would take place, and momentum would be transferred to the lattice through scattering processes leading to energy dissipation.

dissipation. For the cylindrical body, the end result of this process when the entire system has become normal is that there is no more supercurrent flow and no rotation of the body.

Note that the backflow current  $J_x$  is in direction exactly perpendicular to the phase boundary because the forces in the  $\hat{y}$  direction are balanced if the normal state carriers are holes. Because the backflow occurs only over a small boundary layer of thickness  $\lambda_L$  it will give rise to no energy dissipation assuming the mean free path is larger than  $\lambda_L$ . Instead, if the normal state carriers were electrons rather than holes the situation would be as depicted in Fig. 5. There would be no force by the lattice on the electrons and the backflowing electrons would acquire a tangential velocity in the  $+\hat{y}$  direction, and this current would die out by scattering transmitting momentum to the lattice and dissipating Joule heat. Thus within this scenario the observation that no Joule heat is dissipated is only compatible with the normal state carriers being hole-like.

In summary, we have pointed out in this paper that the

fact that the superconductor-normal transition is found experimentally to be reversible, hence occurs without Joule heat dissipation, poses a conundrum that is not addressed in the conventional theory of superconductivity: how does the momentum of the supercurrent get transferred to the superconducting body without energy dissipation? We have pointed out that Bloch's theory of electrons in crystals shows that electrons near the top of the band are effective in transmitting their momentum to the lattice in a reversible way because they undergo Bragg scattering, and electrons near the bottom of the band are not. For electrons moving in crossed electric and magnetic fields we have pointed out that when the Hall coefficient is negative no net momentum transfer between electrons and the lattice occurs, while if the Hall coefficient is positive a net momentum transfer between electrons and the lattice necessarily occurs. Finally, we have proposed a specific scenario using physical elements from the theory of hole superconductivity that explains how the Meissner current stops and the momentum is transferred to the body as a whole without energy dissipation when the superconductor goes normal.

From its inception [16] the theory of hole superconductivity proposed that hole carriers are indispensable for superconductivity to occur. Over the years we have discussed many different reasons in favor of this hypothesis [12]. The additional reason discussed in this paper is arguably the most compelling one.

## Acknowledgments

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